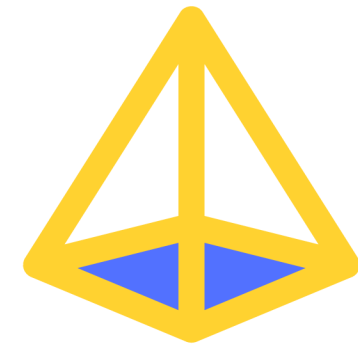
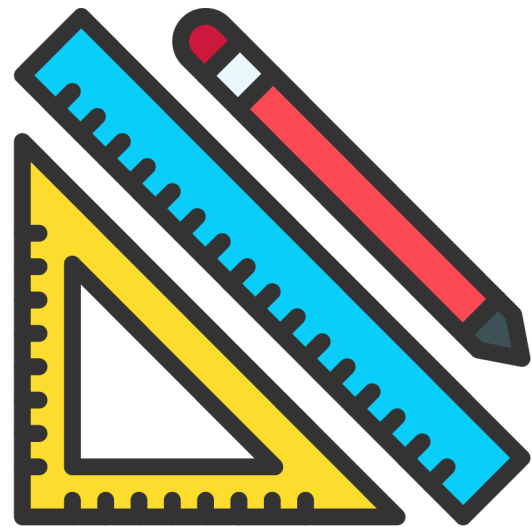
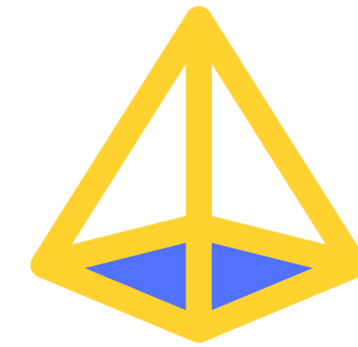


INTRODUCTION



TO



EUCLIDEAN

GOMETRY



EUCLIDEAN GEOMETRY: OVERVIEW

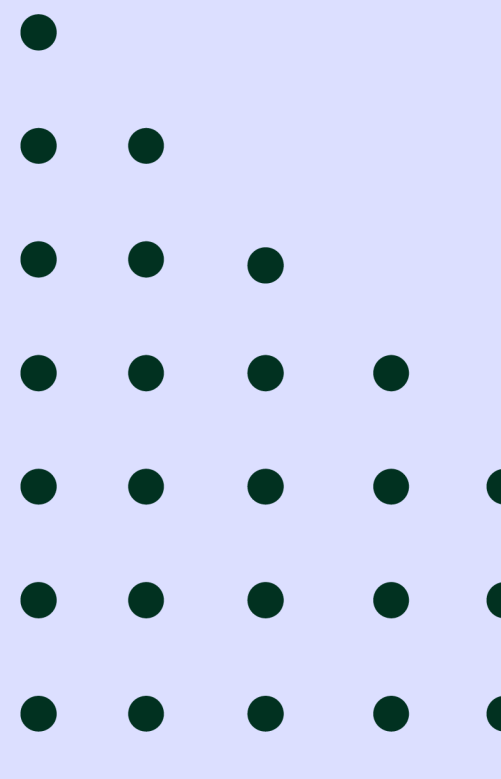

Euclidean Geometry is based on the geometry developed by the Greek mathematician Euclid. It studies shapes, sizes, and the properties of space—primarily dealing with points, lines, angles, triangles, and other polygons—using logical reasoning and proofs.





◆ BASIC DEFINITIONS & POSTULATES

Euclid's Postulates (simplified): A straight line segment can be drawn joining any two points. A line segment can be extended infinitely in both directions. A circle can be drawn with any center and radius. All right angles are equal. If two lines intersect a third line such that the interior angles on the same side add to less than 180° , the two lines will eventually intersect on that side.

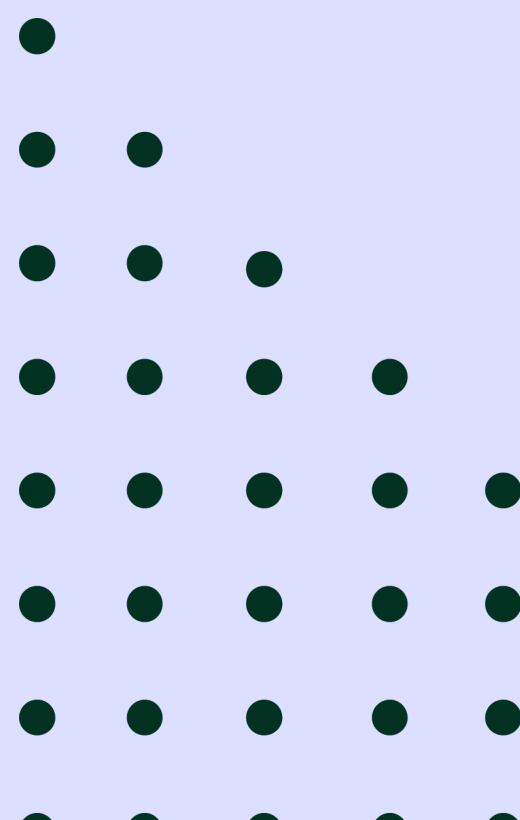


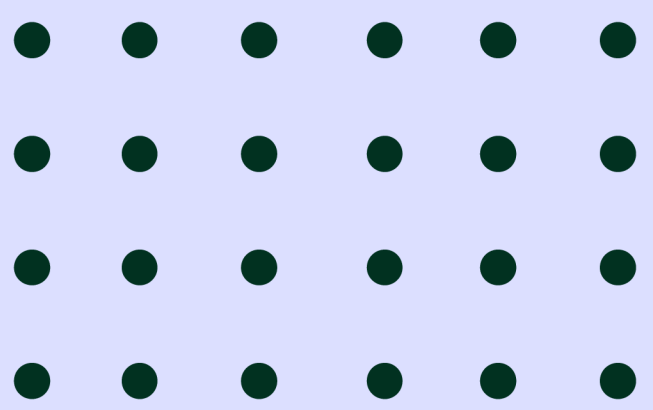
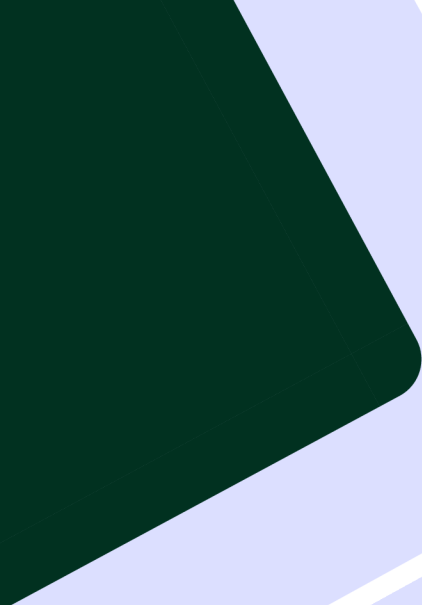
◆ TRIANGLE GEOMETRY THEOREMS

1. The Sum of Angles in a Triangle Theorem: The sum of the interior angles of any triangle is 180° . Proof Idea: Use a line parallel to one side of the triangle to show alternate interior angles.


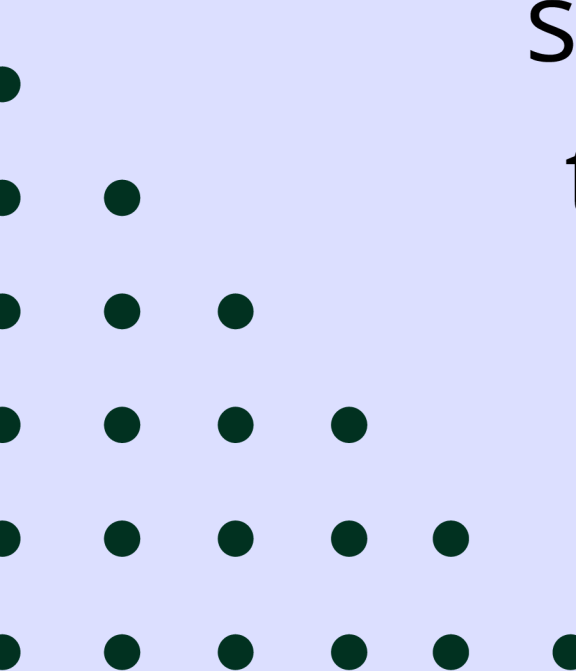
2. Exterior Angle Theorem Theorem: An exterior angle of a triangle is equal to the sum of the two opposite interior angles.

Reasoning: This follows directly from the 180° rule.





3. Isosceles Triangle Theorem Theorem: If two sides of a triangle are equal, then the angles opposite them are equal. Converse: If two angles are equal, the sides opposite them are equal.



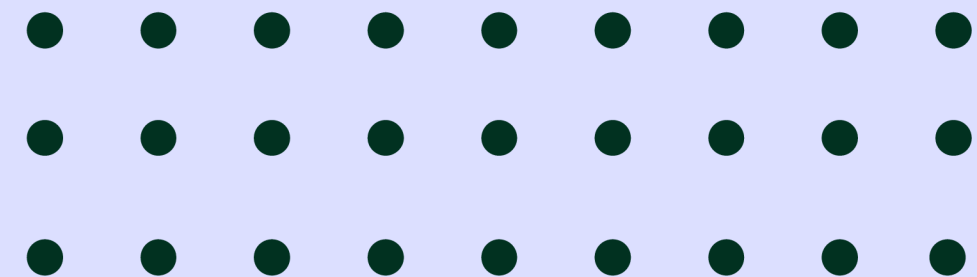
4. Midpoint Theorem Theorem: The line segment joining the midpoints of two sides of a triangle is parallel to the third side and half its length.

◆ PARALLEL LINES AND ANGLES

Corresponding Angles Theorem If a transversal cuts two parallel lines, corresponding angles are equal.

Alternate Interior Angles Theorem If a transversal cuts two parallel lines, alternate interior angles are equal.

Co-Interior (Same-Side Interior) Angles Theorem If a transversal cuts two parallel lines, co-interior angles are supplementary (add to 180°).





◆ CONGRUENCE AND SIMILARITY

Congruent Triangles (SSS, SAS, ASA, RHS): Triangles are congruent if:

SSS: All 3 sides are equal.

SAS: Two sides and the included angle are equal.

ASA: Two angles and the included side are equal.

RHS: Right angle, Hypotenuse, and one Side are equal.

Similar Triangles (AA, SAS, SSS): Triangles are similar if: AA: Two angles are equal.

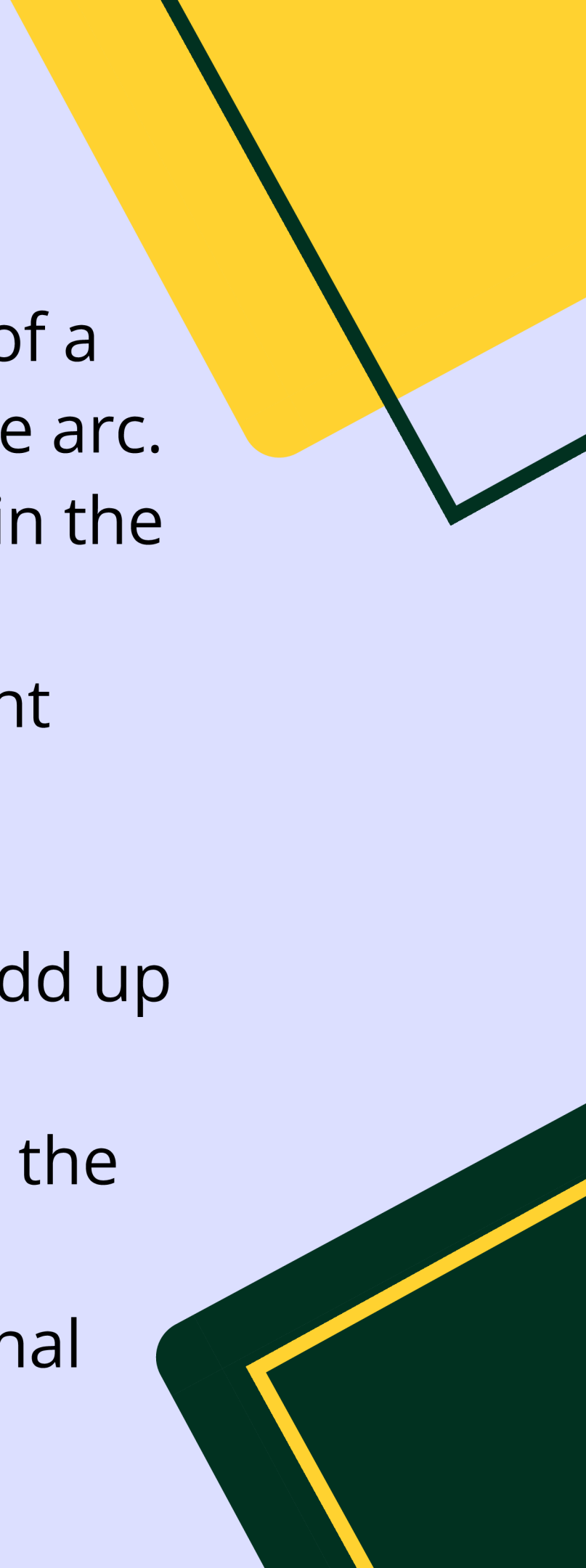
SAS: Two sides are in proportion and the included angle is equal.

SSS: All sides are in proportion.





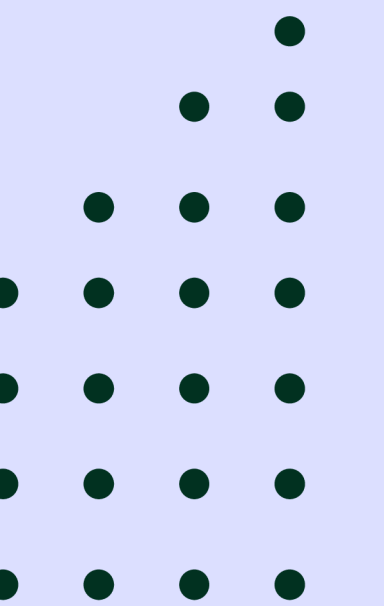
◆ CIRCLE GEOMETRY THEOREMS

1. Angle at the Center Theorem The angle subtended at the center of a circle is twice the angle subtended at the circumference by the same arc.
 2. Angles in the Same Segment Angles subtended by the same arc (in the same segment) are equal.
 3. Angle in a Semi-Circle The angle subtended by a diameter is a right angle (90°).
 4. Cyclic Quadrilateral Theorem The opposite angles of a cyclic quadrilateral (one with all corners on a circle) are supplementary (add up to 180°).
 5. Tangent-Radius Theorem A tangent to a circle is perpendicular to the radius at the point of contact.
 6. Two Tangents from a Point Tangents drawn from the same external point to a circle are equal in length.
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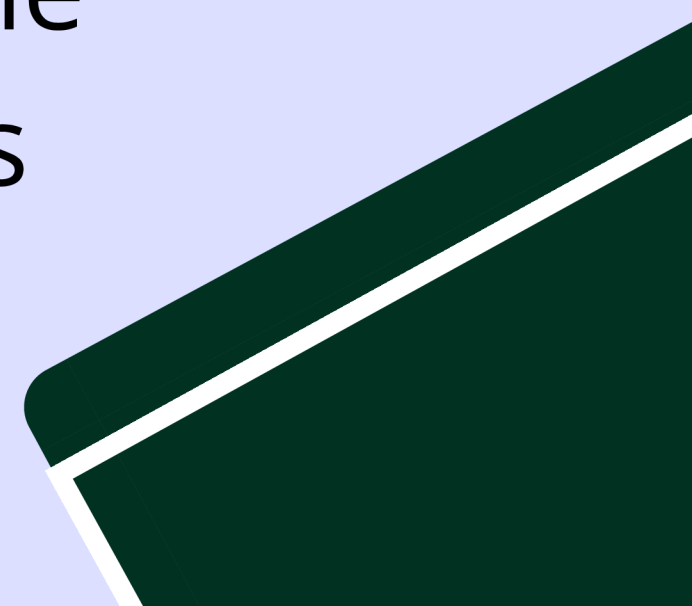


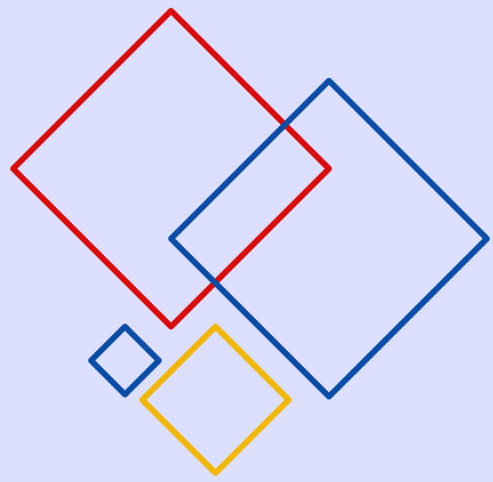
◆ PROPORTIONALITY THEOREMS

1. Theorem of Proportionality (Intercept Theorem) If a line is drawn parallel to one side of a triangle and intersects the other two sides, it divides those sides proportionally.



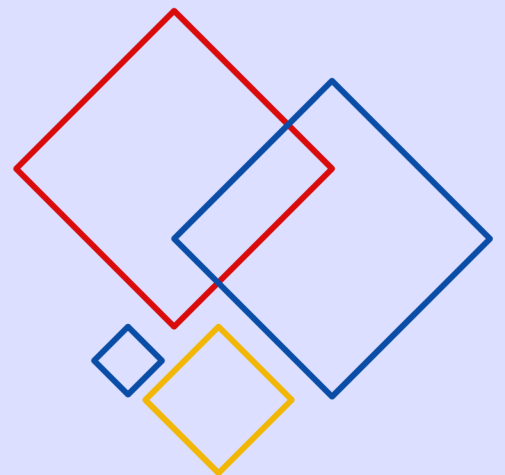
2. Converse of the Theorem of Proportionality If a line divides two sides of a triangle in the same ratio, it is parallel to the third side.





◆ QUADRILATERAL PROPERTIES

- Parallelogram: Opposite sides are equal and parallel. Opposite angles are equal.
- Diagonals bisect each other.
- Rhombus: All sides equal.
- Diagonals bisect each other at right angles.
- Rectangle: All angles are 90° .



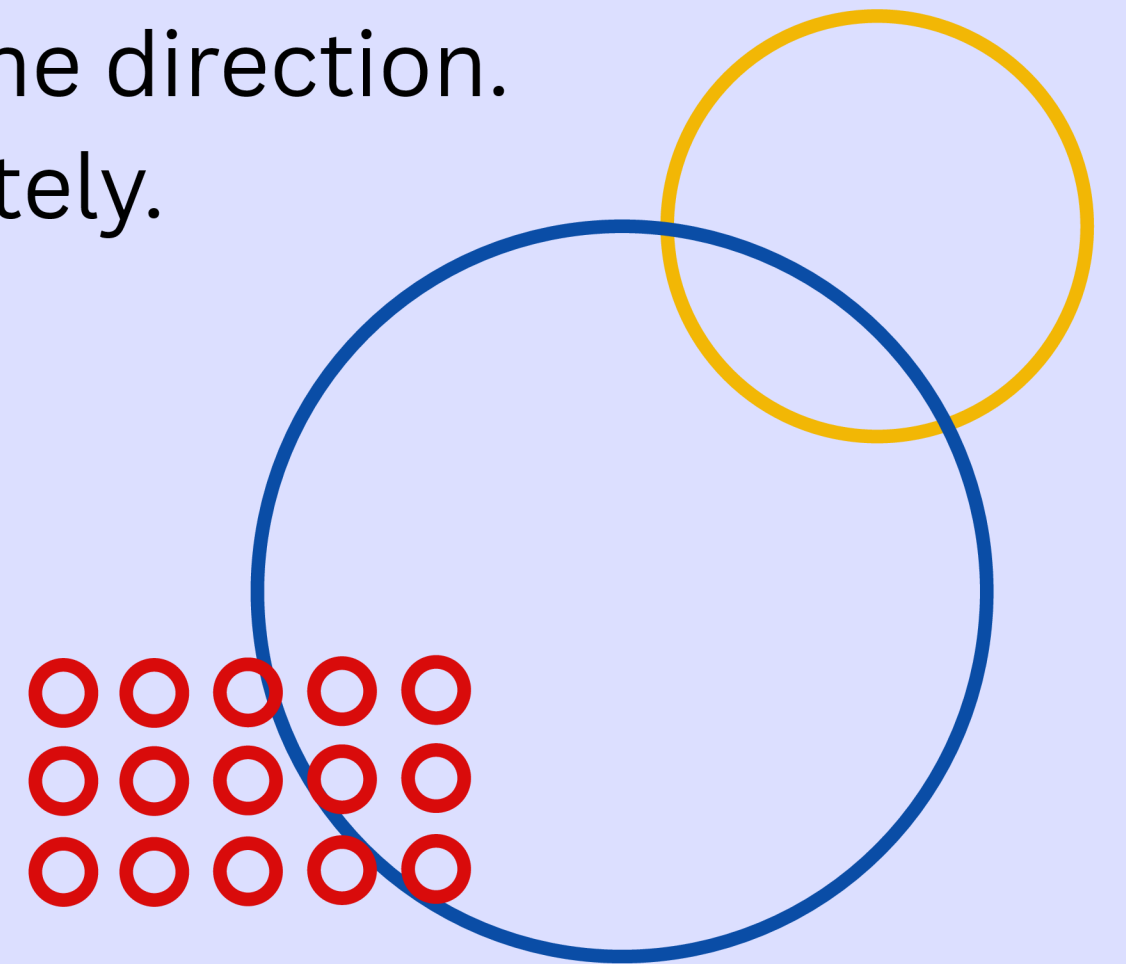
Core Concepts

- **Basic Definitions**

- Point: No size, only position.
- Line: Straight one-dimensional figure with infinite length.
- Line Segment: Part of a line with two endpoints.
- Ray: A line with one endpoint extending infinitely in one direction.
- Plane: A flat two-dimensional surface extending infinitely.

- **Angles**

- Acute: $0^\circ < \text{angle} < 90^\circ$
- Right: $\text{angle} = 90^\circ$
- Obtuse: $90^\circ < \text{angle} < 180^\circ$
- Straight: $\text{angle} = 180^\circ$
- Complementary: $\text{sum} = 90^\circ$
- Supplementary: $\text{sum} = 180^\circ$
- Vertical Angles: opposite angles made by intersecting lines (congruent).



▲ Triangles

Types of Triangles

- **Equilateral:** 3 equal sides, 3 equal angles (each 60°).
- **Isosceles:** 2 equal sides, 2 equal angles.
- **Scalene:** All sides and angles different.
- **Right Triangle:** One 90° angle.

Triangle Theorems

- **Triangle Sum Theorem:** Interior angles sum = 180°
- **Exterior Angle Theorem:** Exterior angle = sum of two remote interior angles.
- **Pythagorean Theorem** (*right triangles*):

$$a^2 + b^2 = c^2$$

Area of Triangle

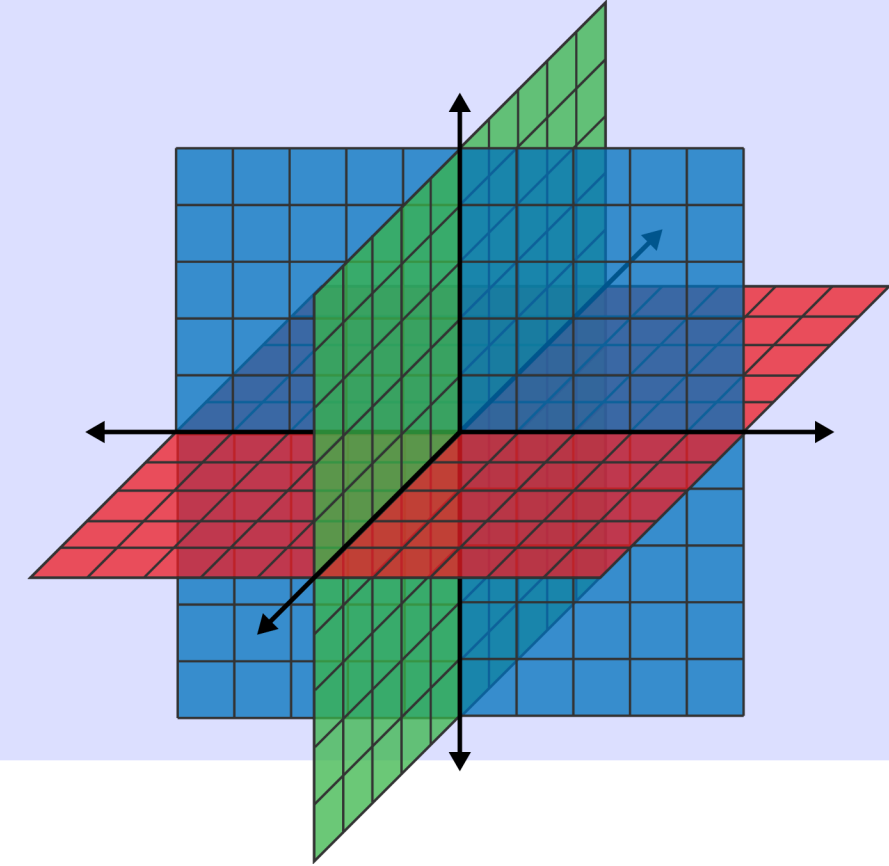
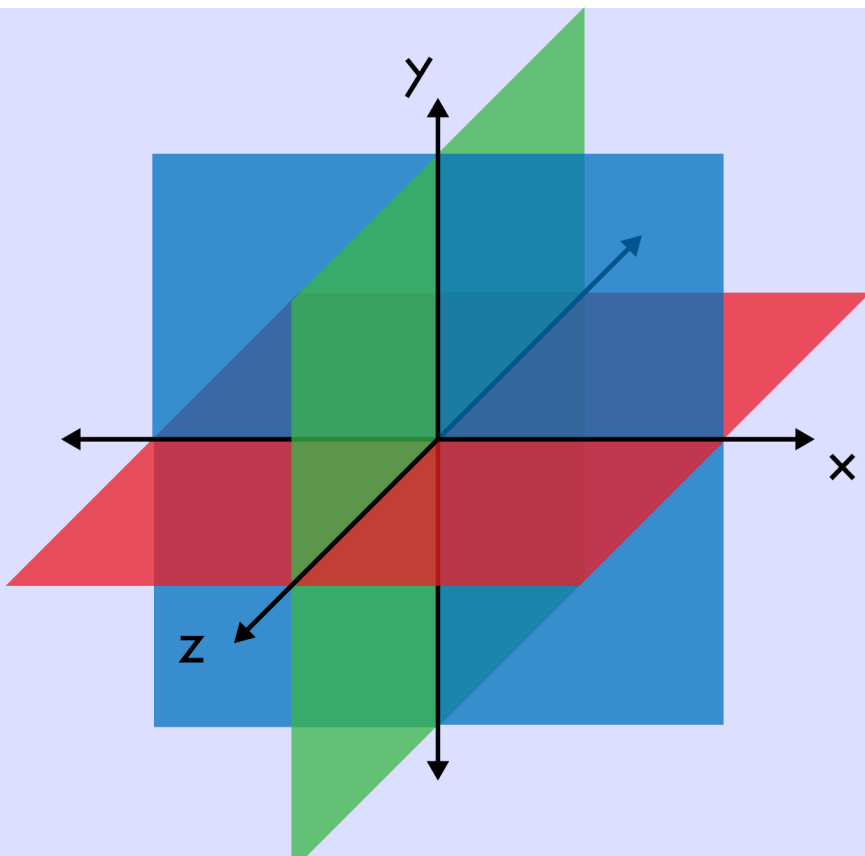
- Basic: $\frac{1}{2} \times \text{base} \times \text{height}$
- Heron's Formula:

$$s = \frac{a + b + c}{2}, \quad A = \sqrt{s(s - a)(s - b)(s - c)}$$

Quadrilaterals and Polygons

Quadrilaterals

- Parallelogram: Opposite sides equal and parallel.
- Rectangle: Parallelogram with right angles.
- Rhombus: Parallelogram with all sides equal.
- Square: Rhombus + rectangle.
- Trapezoid: One pair of parallel sides.



Polygon Angle Sums

- Interior angle sum:

$$(n - 2) \times 180^\circ$$

- Each interior angle (regular polygon):

$$\frac{(n - 2) \times 180^\circ}{n}$$

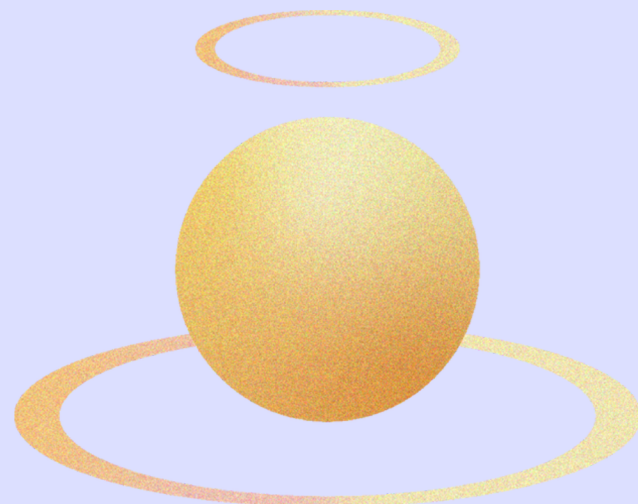
- Each exterior angle (regular polygon):

$$\frac{360^\circ}{n}$$

Circle Geometry

Parts of a Circle

- Radius (r): Distance from center to edge.
- Diameter (d): $2r$
- Chord: Line connecting two points on the circle.
- Tangent: Line that touches the circle at one point.
- Arc: Part of the circumference.
- Sector: Pie-shaped area bounded by two radii and an arc.



Formulas

- Circumference: $C = 2\pi r$
- Area: $A = \pi r^2$
- Arc Length: $\ell = \frac{\theta}{360^\circ} \cdot 2\pi r$
- Sector Area: $A = \frac{\theta}{360^\circ} \cdot \pi r^2$



Coordinate Geometry

- Distance Formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

- Midpoint Formula:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

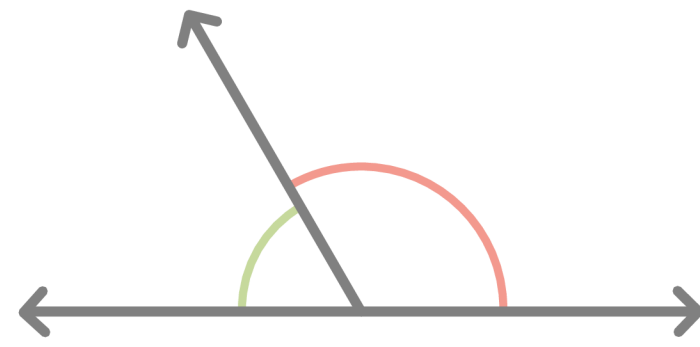
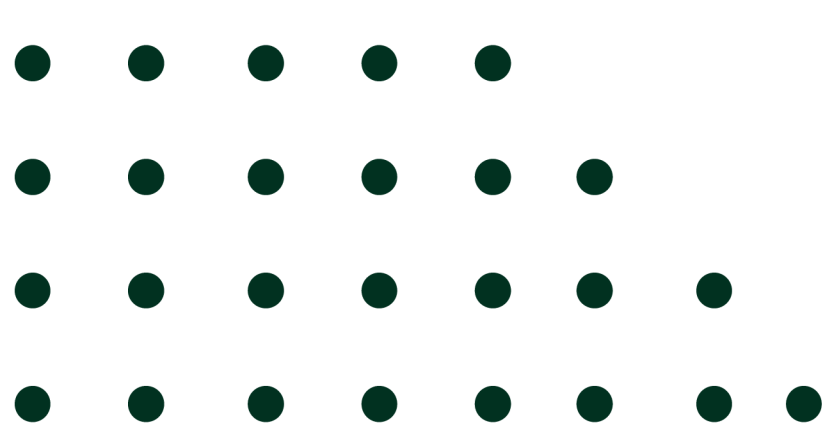
- Slope Formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

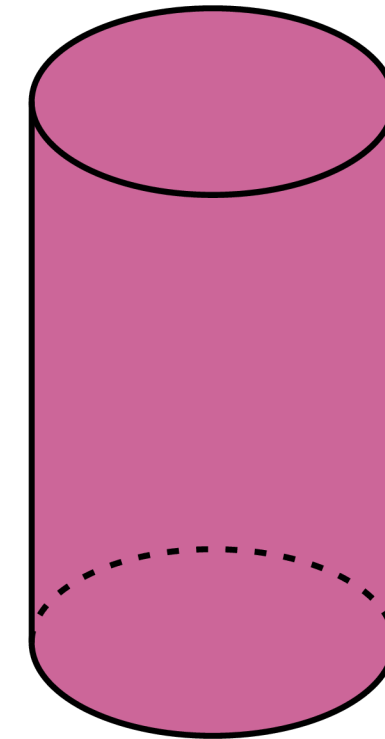
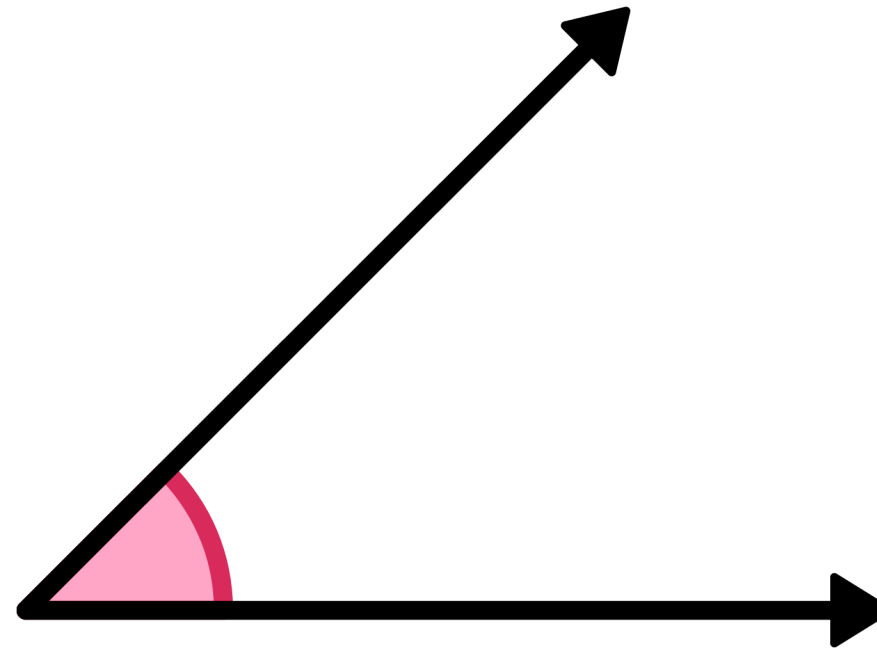
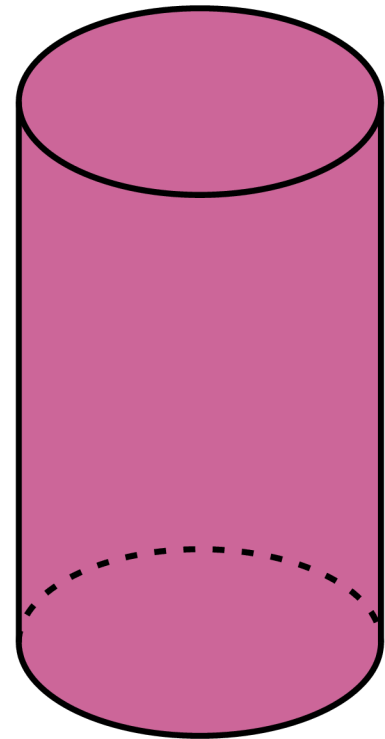
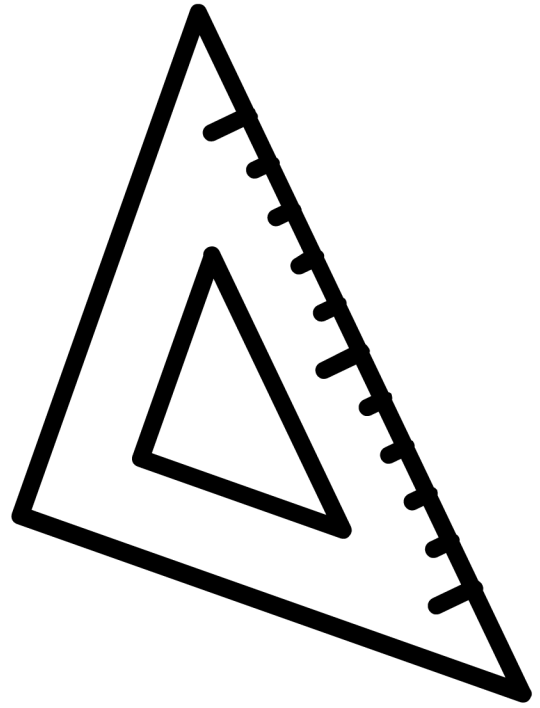


Important Theorems & Postulates

- Congruence (SSS, SAS, ASA, AAS, HL)
- Similarity (AA, SSS~, SAS~)
- Parallel Postulate (Euclidean axiom): Through a point not on a line, there is exactly one parallel line.
- Midsegment Theorem: A midsegment in a triangle is parallel to one side and half its length.



GEOOMETRY IS FUN TO LEARN



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